4. L. I. Sedov, N. G. Vasetskaya, and V. A. Ioselevich, Turbulent Flows [in Russian], Moscow (1974).
5. V. V. Droblenkov and G. I. Kanevskii, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3, 59-65 (1979).
6. V. B. Amfilokhiev, V. V. Droblenkov, G. I. Kanevskii, and N. P. Mazaeva, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3, 40-48 (1981).
7. A. F. Pustoshnyi, Problems of Applied Ship Hydromechanics [in Russian], Leningrad (1975).
8. L. Larsson, Boundary Layers of Ships. Part IV: Calculation of the Turbulent Boundary Layer on a Ship Model. SSPA Rep. No. 47 (1974).
9. W. A. Meyer, AIChE J., 12, No. 3, 522-525 (1966).
10. L. Larsson, Boundary Layers of Ships. Part III: An Experimental Investigation of the Turbulent Boundary Layer on a Ship Model. SSPA Rep. No. 46 (1974).

CONJUGATE PROBLEM OF EVAPORATION IN A LONG CHANNEL AT SMALL REYNOLDS NUMBERS

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The flow characteristics associated with evaporation from the walls of a long flat channel are investigated and the pressure and evaporation rate distribution along the channel are determined with allowance for the redistribution of energy in the walls.

In designing heat exchangers and drying chambers and in constructing models of evaporation from porous bodies it is necessary to consider problems of internal evaporation. A number of features of these problems are manifested in the simple case of evaporation from the walls of a long flat channel.

In the general case, this problem must be examined in the conjugate formulation, with allowance for heat transfer in the walls and the evaporation kinetics, as well as the vapor flow characteristics [1, 2]. In the case of high thermal conductivity of the solid phase and a short slot, for slow flows the fact that evaporation is nonequilibrium in character (allowance for difference between vapor pressure and saturation pressure) may have an important influence on the distribution of evaporation rate along the channel. At low heat fluxes for bodies with a high solid-phase thermal conductivity and low heat of evaporation it is also necessary to take into account the energy redistribution in the walls, which leads to nonuniform evaporation. The evaporation regimes were analyzed and the possible formulations of the problem classified in [2].

If the thermal conductivity of the solid phase is low, evaporation will be "uniform," i.e., the evaporation rate is determined starting from the heat flux supplied. The problem of the sublimation of ice was considered within the framework of this approximation in [3] for a molecular-viscosity vapor flow regime. A method that makes it possible to calculate the pressure distribution along a slot of finite width (with allowance for slip and temperature jump) was proposed and an experimental investigation was carried out. In making the calculations the pressure at the channel outlet was taken from the experimental results. It was shown that the pressure in the slot may be several times greater than the chamber pressure.

However, the initial equations of [3] were written for an incompressible fluid, although the pressure differences are such that the gas must be considered compressible. Evaporation was assumed to be uniform and equilibrium. At the same time, it is useful to analyze the case of nonuniform evaporation, when the energy redistribution in the walls is important, with allowance for the compressibility of the gas.

1. We will consider the conjugate problem of equilibrium evaporation from the walls of a long flat channel with a convergent nozzle at the outlet (width of slot 2h much less than

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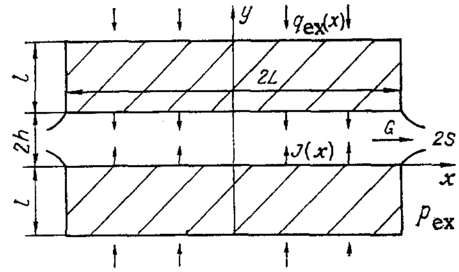

Fig. 1. Scheme of evaporation from channel.

its length 2L, $S = h_{out}/h \leq 1$). Heat is supplied to the outer surface, and outside the slot a constant pressure p_{ex} is maintained (Fig. 1). In the continuum regime the gasdynamic problem is quasisteady and is described by the parabolized Navier–Stokes equations for narrow channels [4] (terms $O(\delta)$, $\delta = h/L \ll 1$, have been discarded):

$$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0, \tag{1}$$

$$\rho(u\partial u/\partial x + v\partial u/\partial y) = -\partial p/\partial x + \partial/\partial y\,(\mu\partial u/\partial y), \tag{2}$$

$$\partial p/\partial y = 0, \tag{3}$$

$$\frac{3}{2}\rho R\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

with the corresponding boundary conditions (the slip velocity and temperature jump are neglected, since when Kn << 1 they give only small corrections):

$$y = 0 \quad \rho v = J, \quad p = p_e(T), \tag{5}$$

$$x = 0 \quad \partial p/\partial x = 0, \quad u = 0, \quad \partial T/\partial x = 0, \tag{6}$$

$$y = h \quad \partial u/\partial y = 0, \quad v = 0, \quad \partial T/\partial y = 0, \tag{7}$$

$$x = L \quad G = \rho u_m = BS\delta p_e(T_0)/\sqrt{2\pi RT_0}\, f(p/p_{ex}). \tag{8}$$

Here, $p_e = p_{e*}\exp(-Q/RT)$ is the saturated vapor pressure; R is the gas constant; $f(p/p_{ex})$ is the gasdynamic flow rate function obtained from the problem of adiabatic gas flow from a nozzle [2, 5].

Since the flow rate is determined by the energy supplied, for the pressure and Mach number at the outlet from the slot we have the estimates [2]:

$$p_{in} \sim \max\{p_{ex}, (q/Q)(\delta S)^{-1}\sqrt{RT}\},$$
$$M_{in} \sim S\min\{1, (q/Q)(\delta S)^{-1}\sqrt{RT}/p_{ex}\} \leqslant 1.$$

From the continuity equation we obtain: $v_* \sim u_*\delta \ll u_*$ (here and in what follows characteristic values are denoted by a star). The momentum equation gives an estimate of the pressure differences along the slot (the difference is estimated from the gradient multiplied by the characteristic dimension):

$$\Delta p/p \simeq O(M^2)(1 + O(1/\mathrm{Re})), \tag{9}$$

where $\mathrm{Re} = \rho_* u_* h^2/\mu L \simeq \rho_* v_* h/\mu \simeq q_* h/Q\mu$ is the Reynolds number for flows in long channels (Hele-Shaw type flows [6]), which for evaporation problems is determined by the width of the slot, the heat flux supplied, the heat of evaporation and the viscosity of the gas. The pressure differences may be fairly large when $M = O(1)$ (i.e., when $S = O(1)$) or when $M^2/\mathrm{Re} \gtrsim 1$. At the same time, the pressure difference across the slot $\Delta_y p/p = O(\delta) \ll 1$ [4].

From the energy conservation equation we have the estimate for the transverse temperature differences in the gas:

$$\Delta_y T/T = O(\mathrm{Re}\,\Delta_x T/T) + O(M^2). \tag{10}$$

2. Let us consider the simplifications to Eqs. (1)-(4) possible as a result of the smallness of Re. When $M_{in} \lesssim \sqrt{\mathrm{Re}}$, in accordancce with estimate (9), the characteristic pressure differences $\Delta p/p \lesssim 1$; the characteristic pressure $p_* \sim p_{in}$. The velocity u is proportional to $(h^2/\mu)(\partial p/\partial x)$, as a result of which the convective terms in the momentum equation (2) are

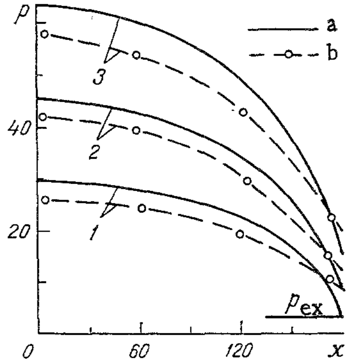

Fig. 2. Pressure in slot p (Pa) as a function of the longitudinal coordinate x (mm) [a) calculations based on (13), b) experiment [3], h = 4 mm, S = 1]: 1) J = $1.8 \cdot 10^{-4}$ kg/m²·sec; 2) $4 \cdot 10^{-4}$; 3) $7.7 \cdot 10^{-4}$.

of the order of M_{in}^2 relative to the viscous terms and in the principal approximation they may be omitted. Correct to terms $O(M_{in}^2, \mathrm{Re})$ the energy equation reduces to $\partial T/\partial y = 0$.

Thus, the pressure, temperature and density of the gas are constant across the slot; the gas velocity has a parabolic distribution, as in incompressible Poiseuille flow, and the flow rate is equal to

$$G = \rho u_m = -\frac{h^2}{3\mu}\frac{p}{RT}\frac{\partial p}{\partial x}.$$

As distinct from the case of Poiseuille flow, the density and temperature will vary along the slot, and the flow rate will also be variable as a result of injection. From the continuity equation and (5), (7) we obtain the equation for the pressure

$$\frac{h^3}{3\mu}\frac{\partial}{\partial x}\left(\frac{p}{RT}\frac{\partial p}{\partial x}\right) = -J(x). \tag{11}$$

The corresponding boundary conditions have the form:

$$\left.\frac{\partial p}{\partial x}\right|_{x=0} = 0, \quad \frac{h^2}{3\mu}\frac{p}{RT}\left.\frac{\partial p}{\partial x}\right|_{x=L} = BSp\delta/\sqrt{RT}\, f\left(\frac{p}{p_{ex}}\right). \tag{12}$$

The pressure distribution is determined by the evaporation rate J(x), which, generally speaking, is found from the solution of the conjugate problem.

When $1 \gg M_{in} \gg \sqrt{\mathrm{Re}}$, in accordance with estimate (9), $\Delta p/p \sim M_{in}^2/\mathrm{Re} \gg 1$, which indicates that the characteristic pressure in the inner region p_* is much greater than p_{in}. In order to estimate p_* we make use of the continuity equation. Considering that the flow rate G varies on a distance of the order of L, we obtain

$$G \sim \frac{1}{RT}\frac{h^2}{\mu}\left(p\frac{\partial p}{\partial x}\right)_{x=L} \sim \frac{1}{RT}\frac{h^2}{\mu}\frac{p_*\Delta p_*}{L} \sim \frac{1}{RT}\frac{h^2}{\mu}p_{in}^2\frac{M_{in}^2}{\mathrm{Re}},$$

whence for $\Delta p_* \sim p_*$

$$p_*/p_{in} \sim M_{in}/\sqrt{\mathrm{Re}} \gg 1, \quad M_* \sim M_{in}\sqrt{\mathrm{Re}} \ll M_{in},$$

where M_* is the characteristic value of the Mach number in the inner region. The dimension of the region of elevated gradients, in which $M \sim M_{in}$, is of the order of L ($\mathrm{Re}/M_{in}^2 \ll L$).

In this region $\partial/\partial x \sim L^{-1}(M_{in}^2/\mathrm{Re}) \gg L^{-1}$; however, $L^{-1}(M_{in}^2/\mathrm{Re}) \sim h^{-1}(\mathrm{Kn}/M_{in})M_{in}^2 \lesssim h^{-1}$. $\mathrm{Kn} \ll h^{-1}$ for $\mathrm{Kn} \ll 1$. Therefore in the continuum regime Eqs. (1)-(4) and (11) are valid over the entire region of flow, despite the increase in longitudinal gradients, and the discarded terms are of the order $O(M_{in}^2)$ in the end region and $O(M_{in}^2\mathrm{Re})$ in the main region of flow.

We will estimate the heat flux in the gas perpendicular to the walls. As follows from (4) and (9), the ratio of this flux to the energy expended on evaporation:

$$q_y/JQ \simeq RT/Q\,(\Delta_x T/T + M^2/\mathrm{Re}).$$

TABLE 1

Curve No.	A	C	ν	$\bar{Q}$	H_p	$\sqrt{A}/C$
1	10^{-6}	0,002	0,1	20	12,5	0,5
2	10^{-5}	0,02	0,1	20	1 25	0,15
3	10^{-6}	0,02	0,1	20	0,125	0,05
4	10^{-6}	0,02	0,1	15	0,095	0,05
5	10^{-5}	0,02	0,01	20	0,125	0,15

Taking into account the estimate $\Delta_x T/T \simeq (RT/Q)\Delta p/p$, we find that for $M_{in}^2/\text{Re}\bar{Q} \ll 1$ the heat flux can be neglected over the entire region of flow. When $\text{Re} \ll M_{in}^2$ this assumption is valid with accuracy $O(\bar{Q}^{-1})$ in the central region, where $M_*^2 \lesssim \text{Re}$.

When $M_{in} \sim 1$ near the nozzle (for $1 - x/L \sim \text{Re}$) there will be two-dimensional flow, and the convective terms in (2) cannot be neglected; the temperature and density will not be constant across the slot. In this case the evaporation in the end region may be intense ($M_{ev} \sim 1$; $(p_e - p)/p \sim 1$), but as a result of the condition $p_* \gg p_{in}$ the evaporation rates J_* and J_{in} will be comparable. If the parameters are suitably averaged, calculations in accordance with the one-dimensional theory of flows with friction but without injection give an error in the pressure determination of 15-25% for $M \sim 1$ [7]. An error of this order has only a slight effect on the solution for $1 - x/L \gg \text{Re}$, where $M_* \sim M_{in}\sqrt{\text{Re}} \ll 1$ and $p_* \gg p_{in}$. Consequently, it is also possible to use (11) with boundary conditions (12) for S = 1.

When J = const the solution of (11), (12) has the form:

$$p = p_{in}\sqrt{3(M_{in}^2/\text{Re})[1-(x/L)^2]+1}. \tag{13}$$

An estimation of the terms discarded in deriving (11) shows that, despite the increase in the gradients when $M_{in}^2 \gg \text{Re}$, Eq. (11) is valid with accuracy $O(M_{in}^2\text{Re})$ in the central region of the slot and with accuracy $O(M_{in}^2)$ in the end region $(1 - x/L) \sim \text{Re}$. Comparison with experiment [3] (Fig. 2) shows that boundary conditions (12) can in fact be used when S = 1. The fact that the calculated curves lie above the experimental ones in the central region can be attributed to the nonuniformity of evaporation and the effect of slip (the treatment of slip by the authors of [3] is incorrect as a result of the determination of Kn from p_{ex}, whereas $p_{in} \gg p_{ex}$).

3. The nonuniformity of evaporation is caused by the cross flow of energy in the walls as a result of the presure gradient along the slot (since $p = p_e(T_0)$, the pressure gradient causes a gradient of the evaporation surface temperature T_0) [1, 2]. A criterion of this redistribution is the condition $(\ell/L)q_x/q_{ex} \gtrsim 1$. In [2] it was shown that for equilibrium ($p = p_e$) evaporation this condition reduces to $H_p \lesssim 1$, where H_p is a dimensionless complex: $H_p = \beta^{-1}(\bar{Q}q_*L/\lambda_s T_*)M_*^{-2}\min\{\text{Re}\ 1\}$.

We will consider the solution of the conjugate problem for equilibrium evaporation (region VI in the classification of [2]) at small Re numbers.

We write (11)-(12) in dimensionless form (retaining the same notation for the dimensionless quantites):

$$A\frac{\partial}{\partial x}\left(\frac{p}{T}\frac{\partial p}{\partial x}\right) = -J,\ A = \frac{h^3}{3\mu}\frac{p_{ex}^2}{RT_*L^2}\frac{Q}{q_*},$$

$$\left.\frac{\partial p}{\partial x}\right|_{x=0} = 0,\ G|_{x=1} = A\frac{p}{T}\left.\frac{\partial p}{\partial x}\right|_{x=1} = C\frac{p}{\sqrt{T}}f(p),\ C = \frac{Q}{q_*}S\delta\frac{p_{ex}}{\sqrt{RT_*}},$$

where G has been divided by $G_* = (q_*/Q)\delta^{-1}$, J by $J_* = q_*/Q$, p by the external pressure p_{ex} (the dimensionless pressure may be much greater than unity), and T by the saturation temperature T_* at p_{ex}. In the steady case for a uniform heat supply G = 1, so that the constant C gives the characteristic values of the pressure at the outlet from the slot. The constant A depends on the viscosity of the vapor and does not depend on S; the ratio $A/C^2 \simeq \text{Re}/S^2$.

In dimensionless variables the gradients at x = 1 are as follows:

$$\frac{1}{p}\left.\frac{\partial p}{\partial x}\right|_{x=1} \simeq \frac{C_*^2}{A},\quad \frac{1}{T}\left.\frac{\partial T}{\partial x}\right|_{x=1} \simeq \frac{C_*^2}{A\bar{Q}},\quad C_* = \min\{C,\ 1\}.$$

In the case of uniform evaporation (J = const) when q = 1 and C << 1 (critical discharge regime, $M_{out} = 1$) the solution of (11) takes the form:

$$p = \sqrt{T/A}\sqrt{(1-x^2) + A/C^2}.$$

The pressure decreases monotonically from $p_{max} = p(0)$ to p(1). The ratio $(p_{max}/p_{in}) = \sqrt{C^2/A + 1} \simeq S/\sqrt{Re} << 1$ for $Re/S^2 << 1$. A change of 15-20% in C has only a slight effect on the pressure in the central region. The "characteristic" pressure difference $(p(0) - p(1))/p(0) \sim 1$, so that $\Delta T/T \sim 1/\bar{Q}$. Thus, in the inner region the characteristic values of the pressure and temperature gradients are much less than at x = 1.

The quantity H_p is expressed in terms of the dimensionless parameters as follows:

$$H_p = (A/C_*^2)(Q\gamma/\beta^2),\ \beta = l/L,\ \gamma = q_* l/\lambda_s T_* \simeq \Delta_y T/T_*,$$

therefore the "nonuniformity" condition $H_p \lesssim 1$ takes the form:

$$AQ\gamma/C_*^2\beta^2 \lesssim 1. \tag{14}$$

This relation holds good in the end region with elevated gradients; in the main region, where $M_* \sim \sqrt{Re}$, the condition takes the form:

$$Q/\gamma\beta^2 \lesssim 1,$$

which for small Re/S^2 is a more rigorous requirement than (14).

The heat conduction equation

$$\beta^2\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 = \partial T/\partial t$$

with boundary conditions

$$\partial T/\partial y|_{y=-1} = -\gamma q_{ex}(x),\quad \partial T/\partial x|_{x=0} = \partial T/\partial x|_{x=1} = 0,$$

$$\partial T/\partial y|_{y=0} = -A\gamma\partial/\partial x\,[(p_e(T)/T)\,\partial p_e(T)/\partial x)],\ p_e = \exp[\bar{Q}(1-1/T)],$$

where time has been divided by $\tau_s = \rho_s c_s l^2/\lambda_s$; x by L; y by l; and q_{ex} by the characteristic flux q_*, was solved numerically. An explicit finite-difference scheme was used, and the temperature at y = 0 was determined from the exponentially nonlinear boundary conditions by the sweep method on each time step. A check by the net densification method revealed that the scheme is stable and gives an accuracy of 1-2%.

The results of the calculations for q_{ex} = const are presented in Figs. 3 and 4. Figure 3a shows the function J(x) at various moments of time. Clearly, although $H_p > 1$, so that in the steady case the nonuniformity of evaporation must be slight, in the stabilization process considerable nonuniformity of J is observed. This is because the rate of variation of the temperature of the evaporating surface is essentially different for different x, as a result of which the temperature gradients are greater than in the steady case (cf. Fig. 3b) and hence the nonuniformity is also greater. The pressure distribution for these cases is shown in Fig. 3b.

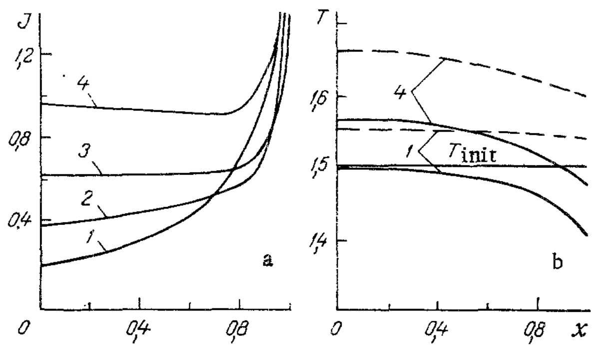

Fig. 3. Flow rate (a) and temperature (b) as functions of the longitudinal coordinate during approach to the steady-state regime ($A = 10^{-6}$, $C = 2\cdot10^{-3}$, $\gamma = 0.1$, $\beta = 0.2$, $H_p = 12.5$, $\sqrt{A}/C = 0.5$): 1) $\tau = 0.2$; 2) 0.6; 3) 1.2; 4) 10. Continuous curve, T_0; broken curve, T_{ex}.

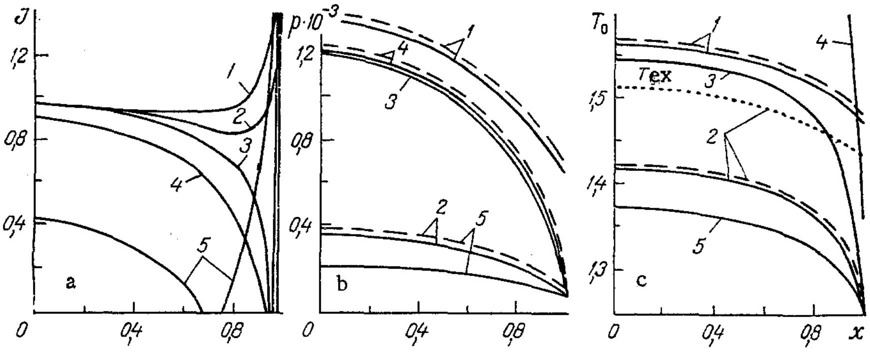

Fig. 4. Flow rate (a), pressure (b), and temperature (c) as functions of the longitudinal coordinate in the steady-state regime. Broken curves are for J = const, the dots represent T_{ex}. The curves not indicated in Fig. 4a are 3 and 4 respectively. The various values of the parameters are given in Table 1.

In Fig. 4 we have reproduced the results of the calculations in the steady-state regime for various values of the leading parameters. Clearly, the nonuniformity of evaporation is the greater the smaller the value of H_p. The dimension of the zone of strong inhomogeneity is of the order of $\sqrt{A/C_*}$. The nonuniformity of evaporation has only a slight effect on the temperature distribution (Fig. 4b) and a marked effect on the pressure distribution when $H_p \lesssim 0.5$, the value of p_{max} being less than in the case of uniform evaporation.

The dependence J(x) is characterized by a minimum in the inner region, and in the case of strong nonuniformity even condensation is possible (Fig. 4, curves 3-5). In all cases a flow rate maximum is observed at the outlet from the slot. As the thermal conductivity of the solid increases (γ decreases), the minimum is displaced toward the center, so that in the limit of isothermal evaporation it lies at the center itself.

The appearance of a minimum on the J(x) curve can be explained as follows. It is clear from relations (11)-(12) that for fixed A and J the smaller the pressure the greater the pressure gradient and hence the temperature gradient. The results of the calculations also show (Fig. 4c) an increase in the gradient $T_0(x)$ with distance from the center. The greater the temperature gradient along the slot the greater the amount of energy transported in the longitudinal direction and hence, for a constant supply of energy from the outer surface, the smaller the heat flux "spent" on evaporation. As a result, with distance from the center the flow rate decreases.

However, in the steady-state case all the energy supplied to the solid must go towards evaporation; at the same time, at x = 1 the end surface is adiabatic and does not evaporate. The energy flux flowing from the central to the end region goes towards evaporating the material near the end of the slot where J has a maximum. The area under the J(x) curve is equal to unity. As a result of the longitudinal heat transfer the transverse temperature gradient at first decreases and the $T_0(x)$ and $T_{ex}(x)$ curves approach each other (Fig. 4c), but then as a result of the increase in energy supply to the outlet region it increases, the $T_{ex}(x)$ curve falling less steeply than $T_0(x)$.

The solution depends on the dimensioness complexes A, C, γ, etc. For fixed values of the dimensionless parameters the values of Re and $M_{in} \sim S$ may be different, so that for small S the results are valid correct to S << 1 throughout the entire region.

When S ~ 1 in an end region of dimension of the order of A/C^2 the solution gives the pressure and flow rate only in order of magnitude; consequently, for strong nonuniformity ($\Delta J/J >> 1$) the fall in flow rate in the central region is also given only in order of magnitude; in this case we obtain a qualitative picture of the J(x) and p(x) distributions.

Thus, the calculations confirm the validity of the estimates made in [2] for steady-state evaporation; in the unsteady case the nonuniformity of evaporation has only a slight effect on the evaporating-surface temperature distribution, so that this temperature can be calculated quite accurately for J = const. However, the nonuniformity of the flow rate must be taken into account in designing heat exchangers and in selecting drying regimes.

NOTATION

h, channel width; L, channel length; $\delta = h/L$, the relative channel width; ℓ, thickness of the walls; $\beta = \ell/L$, relative wall thickness; S, relative area of the outlet cross section; p, gas pressure; p_{ex}, pressure outside the slot; p_{in}, pressure at the nozzle inlet; p_e, saturation pressure; T, temperature; T_0, evaporation surface tenperature; Q, heat of evaporation; $\bar{Q} = Q/RT$, dimensionless heat of evaporation; μ, gas viscosity; λ, gas thermal conductivity; λ_S, solid-phase thermal conductivity; q, heat flux; M, Mach number; Re, Reynolds number; Kn, Knudsen number; J, evaporation rate; and G, mass flow rate per unit slot area.

LITERATURE CITED

1. V. I. Nosik, Proc. IX Conference of Young Scientists FALT Moscow Physicotechnical Institute, Dolgoprudnyi, 1984 [in Russian], Moscow (1984), pp. 4-9.
2. V. I. Nosik, Inzh.-Fiz. Zh., 52, No. 1, 116-122 (1986).
3. P. A. Novikov, G. L. Malenko, and L. A. Lyubin, Inzh.-Fiz. Zh., 26, No. 1, 58-63 (1974).
4. J. C. Williams, AIAA J., 1, No. 1, 186-195 (1963).
5. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Moscow (1978).
6. H. Schlichting, Boundary Layer Theory, McGraw-Hill (1966).
7. E. Emmonds (ed.), Fundamentals of Gasdynamics, Princeton (1958).

COLLAPSE TIME OF VAPOR BUBBLES

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UDC 532.529.5

A formula is obtained for the collapse time of a vapor bubble with sharp pressure increase, taking account of the heat transfer between the bubble and the liquid.

Consider the action of a sharp change in external pressure from p to $p_0 + \Delta p$ at a spherical homogeneous vapor bubble in an infinite liquid medium. If Δp is not too large, the process of pressure equalization in the system and establishment of thermodynamic equilibrium inside the bubble, including the vapor–liquid interface, occur relatively rapidly [1-3]. The heat transfer from the bubble to the liquid determines the condensation rate of the vapor, and the time dependence of the bubble radius may be found by solving the system of equations [1, 4]

$$\frac{\partial T}{\partial t} + \frac{dR}{dt}\frac{R^2}{r^2}\frac{\partial T}{\partial r} = \frac{a}{r}\frac{\partial^2 (Tr)}{\partial r^2}; \tag{1}$$

$$T(R, t) = T_0 + \Delta T;\ T(\infty, t) = T(r, 0) = T_0;$$

$$\frac{dR}{dt} = \frac{c\rho_L a}{h\rho_V}\left.\frac{\partial T}{\partial r}\right|_{r=R};\ R(0) = R_0. \tag{2}$$

Here T(r, t) is the spherically symmetric temperature field in the liquid ($r \geqq R$). The vapor temperature is constant over the time of collapse, and is $T_0 + \Delta T$, where ΔT is the change in boiling point of the liquid with increase in pressure by Δp. The collapse time t_c is determined by the condition $R(t_c) = 0$.

The well-known – see [1], for example – estimate of t_c is obtained on substituting the temperature field into Eq. (2) in the form $T = T_0 + \Delta T\ \mathrm{erfc}[(r - R)/2(at)^{1/2}]$, i.e., an accurate solution of the plane thermal problem. This estimate is expressed as follows

$$\tau_c = 4\mathrm{Ja}^2 a t_c/\pi R_0^2 = 1. \tag{3}$$

Here $\mathrm{Ja} = c\rho_L\Delta T/h\rho_V$ is the Jacob number, which is the only defining criterion of thermohydrodynamic similarity in Eqs. (1) and (2) (for water at 100°C, $\mathrm{Ja} \approx 100\ \Delta p/p_0$).

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